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MESSAGE-BASED SEMANTICA FOR SAT PROBLEMS

Abstract. This paper outlines the logical system of MSSG-logic originally developed by the author to represent a particular theory in analytic philosophy of language. That was a theory by philosopher Paul Grice that proposed to consider the intention of the speaker as an important context for the definition of the true meaning of his speech act. MSSG-logic proposes to use special sets of symbols for the additional interpretation of the logical formulas as well as the supersets of these sets ("trees of messages") with defined algebras on them for more complex cases. This particular paper proposes to modify the "linguistic" MSSG-logic to represent the famous computer science boolean satisfiability problems (SAT) with the similar tools. The consequences for computer science and philosophy of language are also discussed.
Keywords: *SAT, boolean satisfiability, computer science, logic for CS, Paul Grice, formal language, analytic philosophy of language.*

Introduction. Author initially proposed message-based logical semantics for formal study of natural language. The main inspiration for the idea was concept by analytic philosopher H. Grice. He researched linguistic semantics and proposed that to define the true meaning of the utterance (speech act) we should take into consideration the intention of the speaker (Grice, 1972, p. 3).

The idea is next. Whatever the words’ meaning "by vocabulary" is, a speaker might be using the speech act to manipulate his audience. Speaker can lie, manipulate the facts or make a certain wrong emphasis in his words. In this way, the true meaning of the speech act is not its plain meaning but the intention of the speaker combined with the "usual" meaning. More generally, the key context of the utterance defines the meaning of the utterance as well as the "vocabulary".

The idea of the message based semantics is adding to the signature of the logic special symbol sets – messages. Messages will define meta-rules for interpreting the semantics of the language studied by the particular logical system. Thus, the message may state that the formal expression is not true because the speaker is lying to achieve his goals etc.

Let us take the obvious example. Politician A. makes an election speech and he needs to introduce the facts about economics in the context he needs rather than just objective facts. We combine the statements in his natural

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language speech with a message: "A. needs facts in a certain context and he may be lying". Interpreter of the speech will have the meta-rule for assessing this speech.

He may use this meta-rule to interpret A.'s speech as a rhetorical speech or he may interpret it differently, according to his own information and purposes. He can try to expose A or just observe the situation based on his own goals. That is the strong side of meta-rules – with some specifications they can be used for a wide range of different purposes. From rhetoric to mathematics.

In this way, messages are wider as a concept than just this narrow usage in analytical philosophy and linguistics. It is possible to build a system of messages that cooperate with each other by rules of a special algebra and represent structure other than just that of natural language. It is especially important for formal languages. For the case of the linguistic application the author proposed to use the abstract systems of messages ("message trees") which later can be developed into an algebraic mathematical structure. It is not obligatory that there should be just "trees". Different algebras and rules of cooperation can present different hierarchies of meta-rules – both vertical and horizontal.

One of these possible alternative applications is computer science SAT (Boolean satisfiability) problems (Manquinho & Marques-Silva, 2004, p. 354). SAT problem is defining the semantics status of the propositional logic formula. It usually concerns the very complex propositional formulas that define certain abstract computational structures which can relate both to the computer hardware design and applied mathematics engineering problems. In this way, whether it is possible to find such a combination of truth values for the parts of the propositional formula so it will be TRUE (satisfiable) is a question which has consequences for theoretical computer science (existence of the appropriate algorithm) and pure hardware circuits design. Sometimes the truth status of such formulas also has consequences for certain problems in an algebraic complexity theory. As a result, the paper proposes to introduce a slightly modified version of this linguistic logical semantics that can help represent the SAT problems.

Analysis of recent research and publications. The connection of the philosophical (non-classical) logics and computer science is quite a hot topic in the scientific discourse of the recent years. Most of these discussions include computational logic i.e. specific logical systems for formal deductive reasoning about the computational functions which are the foundation of any computer system.

From the beginning of the XXth century an extremely important segment of mathematics connected to logic and computer science was automated theorem proving (Biere et al.). Though, Hilbert's Problem seems to be unsolvable there is always progress to make in applying computers to finding new formal results in mathematics.

Another extremely influential topic is temporal logic (Clarke et al.). Research in this area allows for new insights on saving computational powers. In this way it is connected to dynamic logic (Goranko et al.).

In general MSSG-logic version, presented in this particular paper it goes specifically about the SAT problems, but in general it shares many common topics with computational logic and computer science.

The purpose of the article is to research the features of message-based semantic for sat problems

Formulation of the main material.

1. The semantics in the nutshell

The initial semantics states the existence of the additional sets of symbols in the signature of the classic formal logic. Each message will define a certain context for the propositions in question. In this way each such eligible message will be a meta-rule for the particular formal logic.

The initial example of the text-based message (meta-rule) was given in the introduction section. Text may constitute an extremely wide range of topics and tools for defining the context of the propositional formula in question. In a sense, the concept of "messages" in formal logic seriously widens its application domain.

What is the example of such a meta-rule for a non-linguistic usage? Suppose we have a complex statement which involves many different propositions connected by logic operators. There are so many different propositions that it is impossible to build a truth table or use any other classic way to define the status of this statement as the resulting calculation will be unbearably complex.

Suppose also that we have some meta-information about this statement. Among the propositions of the statement there is a proposition "p" and our meta-information states that "p" is false. Introducing this meta-information into the assessment of the initial complex proposition will help a lot with the task.

Assume also that "the message" we introduce gives a general criterion why "p" is false rather than just stating its status. In this way the meta-rule will state that all the propositions of the "p-type" are false. The problem here is to define what is exactly for the proposition of being of "p-type" but it will concern particular engineering problems. If we return to an example with the politician, we can generalize the meta-rule used in this example for all the politicians and their speeches. "You should be cautious in assessing the speeches of the politicians who strive to win the elections, as they can lie to achieve their goals". Then we have a logic that can be useful in the field of rhetorics as well as political science (Graham, 1928, p. 26).

If we would had to assess such contexts formally without the specific messages we would have built a separate logical system modified to represent particular political rhetorical speeches. That would be another interesting project but it should be taken into consideration that message-based logic already allows such contexts as well as the wider range of alternatives without deepening the technical aspects of the formal logic.

This is a very simple and plain example and we can devise something much more complex and subtle but it illustrates the concept. We can develop it further, stating that we have a special curve function that distributes probabilities of certain propositions in the long complex formula being true or false. This particular function may be itself of a certain type and special rules for particular classes of similar functions in the meta-rules may be defined as well.

The "p-type" case may seem like a very unnatural example. It is because the illustration is very basic. It is doubtful that in a real practice the computer scientist will have the evidence that a particular proposition for his logical ventile formal description is false. Or, at least, he will not need the guidance of specific formal structures to use this kind of information. However, there are cases when there is additional non-mathematical (or partially formalized) knowledge about the formal problem including the circuits' design and our semantics gives a chance to use it in solving the problem. In all the cases the information may be codified using textual description in natural language.

Mathematical formalisms thus constitute a little bit more narrow field of application. But there is always a possibility to synthesize approaches by using both formalism and natural language (Wybraniec-Skardowska, 2020, p. 190).

As for the function curves, methods of mathematical calculus are sometimes used to help with the SAT problems and similar mathematical problems that involve propositional formulas. The same goes for the different combinatorial formulas which are actively used in different fields of computer science and applied mathematics. It is difficult to find the particular example that will perfectly fit the description but there is a class of problems for which it would be useful to create a message-based logical system. In fact, the "curve case" is the same as the "false p" case, just more advanced.

Of course it is easy to imagine that for each such complex statement there is more meta-information and more meta-rules accordingly. And these messages can interact with each other so it is not a good decision just to compose one big complex message from them using the "and" operator. Apart from that there could be a cooperation of meta-rules of different types. Some may be probabilistic curves while others are defined facts about propositions et cetera.

Because of this reason, the original paper devoted to Gricean linguistic philosophy proposes to consider more abstract and complex structures instead of the messages alone – already mentioned "trees of messages". Let us suppose there are several meta-rules for one very big propositional formula and they may contradict each other or change meaning when combined. It seems that the set of such messages forms a particular "tree-like" structure.

Then we must define the hierarchy of these messages and, more generally, algebraic relations between different messages. Different "trees" can also influence each other and the case for interaction of different complex propositions gives even a more complex picture. So there will be an algebra for relations between the trees as well as for the relations between the different messages in one single tree (Bjarni, 1984, p. 301).

The formal definition of the logical system remains mostly the same as for the linguistic case. The sets of messages are added as well as connected supersets of trees with the defined functional relations. Then the systems of supersets of trees are connected with appropriate functional relations.

Let us just site the formal definition from the author's original paper on the linguistic variant of the semantics: "We add next elements to the model of the classic propositional logic – [A, T, K, G] where A is a superset of sets of textual symbols composing messages, T is a superset of subsets of A constituting the "message-trees", K is a superset of sets of ordering relations on T constituting algebras for "message-trees" and G is a global algebra for cooperation of different trees (or a blank set if there is no global algebra in the system). Different other types of this semantics are possible through specification of local and global algebras" (Petik, 2022).

SAT case requires almost the same specifications. It just should be mentioned that "A" constitutes not textual messages with meta-rules but additional mathematical formal expressions such as mentioned probability distribution curve or other specifications with additional information about the complex propositional formula in question.

"T"-expression for the "message-trees" remains almost the same. The algebra of the relations between the "trees" and the "messages" inside the "tree" should be more strictly defined than for the linguistic case. The same with the different types of modifications for the semantics. It all can be done through added specifications.

The hierarchy of the messages inside one of the "trees" was mentioned previously. It was also specified that it can be "vertical" or "horizontal". Vertical hierarchy means there are always some prior messages that if in conflict with the inferior ones, cancel them. Horizontal hierarchy is the set of rules for dealing with the messages where it is impossible to understand the priority status. In fact, such a "horizontal" set of rules will be a sort of the algebra for the relations between different messages in the system of a one single tree.

It is possible that such a "horizontal" algebra will involve the probability ranges for the different truth statuses of the propositional formulas. The simple or more advanced probability theory schema can be introduced into the system of formal logic to represent that. It was also mentioned that a certain system of the relations between different trees of messages is also possible to establish. Computer science field of application may involve cases when very complex propositional formulas interact with other complex propositional formulas creating a very hard case for the classic logic to represent.

If there is such a practical case, then the message-trees of both formulas will be interacting. We can devise a similar "global" algebra for these interactions. It may seem that there should be a certain ordered hierarchy between "trees" as well. Some trees will be superior to others. It is easy to formalize this relation as well by the ordering relation on the superset of all the message-trees. That is what is meant by "cooperation of the trees".

2. Consequences for computer science.

The proposed semantics allows for building a logic system that will be more effective in solving the SAT class of problems. This is not a universal solution for this type of problems but being properly implemented it may help with the general approach. As was implied in the introduction paragraph, message-based systems make it easier to specify particular formal logic for some technical problem. If not for this solution, you have to build a new independent specific logic for each local case.

SAT problems are important for the circuits design and have theoretical implications for the algorithmic complexity chapter of the computer sciences. The existence of the MSSG-semantics and its applicability to the technical computer science problem proves the intrinsic relation of natural language, formal systems and computational elements design.

The idea of this paper was devised before the introduction of ChatGPT and the massive spread of the artificial intelligence systems which are based on machine learning. However, the paper was completed after AI became very popular. Despite their usefulness, ChatGPT and similar systems cannot solve a lot of existing mathematical problems including the complex propositional formulas and SAT. In this way, the AI does not cancel the usefulness of the formal systems proposed in this paper (Russel & Norvig, 2010).

More to that, the principle of the AI – learning from textual information and answering to the requests in natural language – seems to imply that it is a particular technological bridge between the study of natural language, programming and formal languages. This will also involve contemporary philosophy of language. Do the classical problems and approaches posed by Wittgenstein and his followers change their status due to the new ways to interpret and understand language? That is a very deep question that concerns philosophy first of all. In general this idea is in mathematical logic/combinatorics and as such is an interesting new approach to some of the classic problems and formal structures in computer science.

3. Consequences for linguistics and philosophy of language.

Of course, semantics of such type as described in this particular paper has its consequences for the linguistics and philosophy of language. The deep connection to the philosophy of language in a new AI-related context was already stated. MSSG-logic originally was created to represent one of the concepts of the philosopher of language in analytical tradition of Paul Grice.

The modification of this semantics presented here proves that there is a deep connection between the formal structures of the natural language and formal languages. That is a known idea in philosophy of mathematics but here it is studied in one more practical aspect. It can be connected to linguistics practical research. MSSG-logic can also be used to study other concepts and theories in the field of philosophy of language. These possible applications include the classic ideas of Ludwig Wittgenstein as well as the later ideas by Grice and his followers and probably some further specifications for the French continental philosophy of language.

Citing the original paper once more: "As was mentioned before, messages as a concept are wider than just the intentional theory of meaning of Grice. Text messages may express more than intention – as meta rules their domain is context of the formal expression as a whole. The domain of message-based semantics is about particular specifications. In this chapter we will outline what such specifications are needed to fit into Gricean theory and theorize on further specifications for the case of computer natural language processing.

Textual messages are actually also wider than meta-rules. Textual information may constitute almost any kind of idea. However, the scope of formal logic presupposes meta-rules as the main content for the messages. Any other kind of textual information would make the relations between different trees chaotic. It will be impossible to build a rigorous algebra.

Though, it is still interesting how such a chaotic system will look like and what will be its parameters. Unrigorised textual information will not define the precise values for the truth function. Instead the influence will be chaotic and extremely complex. In fact, it will be more like the cooperation of two different text messages or pieces of fiction text than mathematical expression. In this sense unrigorised message systems are very promising for natural language processing and artificial intelligence " (Petik, 2022).

The main detail here is about chaotic text systems. It seems that natural language is such a chaotic system. Or it is the system with so many complex rules and local cases that it seems chaotic to the most advanced human mind. That is the type of text that is studied by philology and philosophy of language. It is also in line with the complex interacting tree-based structures.

Compare it to the continental philosophy understanding of the natural language. French structuralism and post-structuralism philosophers studied the language as the center of human culture and introduced the notion of postmodernism and hypertext. That would be a great field for formalization for message-based logic in particular.

MSSG-logic has a lot of consequences for the philosophy of language, both analytic philosophy of language and its continental counterpart. In this way, research in this field will be quite fruitful for the new understanding of the natural language.

Conclusion. This paper proposes to modify the existing linguistic logical semantics to be used for the computer science problems known as the SAT problems. SAT problems are important for algorithmic complexity theory, circuits design and computer science in general.

The semantics requires only a few little details of modification which implies the inner connection of the ordinary language and formal languages used for stating mathematical problems.

This semantics is not the universal tool for solving the problems but it may help with the general approach to such types of problems. The paper will be interesting for computer scientist, logicians, linguists and philosophers interested in computation theory.

Conflict of Interest and other Ethics Statements
The author declares no conflict of interest.

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Ярослав ПЕТІК

ЛОГІЧНА СЕМАНТИКА НА ОСНОВІ ПОВІДОМЛЕНЬ ДЛЯ SAT ПРОБЛЕМ

Анотація. В статті проведено огляд логічної системи MSSG-логіки, спочатку розробленої автором для представлення певної теорії в аналітичній філософії мови. Цю теорію запропонував філософ Пол Грайс, який пропонував вважати намір власника мовлення важливим контекстом для визначення справжнього значення його акта мовлення.

MSSG-логіка пропонує використовувати спеціальні набори символів для додаткового тлумачення логічних формул, а також надмножини цих наборів («дерева повідомлень») із визначеними алгебрами для більш складних випадків. Стаття пропонує модифікувати «мовну» MSSG-логіку для представлення відомих булевих задач задоволення умов (SAT) у галузі комп’ютерних наук за допомогою подібних інструментів. Також обговорюються наслідки для комп’ютерних наук та філософії мови.

Ключові слова: *SAT, boolean satisfiability, комп’ютерні науки, логіка для комп’ютерних наук, Пол Грайс, формальна мова, аналітична філософія мови.*

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